Introduction

How to use this Book - Please Read

The following text combines the notes of the course into a workbook style environment. Whether you are reading and using this text as an eBook or the paper version, it is important to **read everything** and to understand it. One way of doing this is to summarize in your own words and take notes in the margin while you are reading and during class discussions. In some cases, there will be more extending information. It will be marked with a "*More...*". This will help you to better understand the concept and its relationship to other concepts, but is beyond the essential core material for that particular term/concept.

All questions posed within the text should be answered. These questions are designed to help you to **understand and retain the essential information**. These are designated with a Ⓠ . If a question designated with a Ⓠ is easy, then answer the advanced question designated with an (AQ) . Not all questions will have an (AQ) . It is also important to take your time with the Explorations, whether they are done in class or at home. They are designed to give you a deeper understanding of the information (contained within) by finding it on your own. You will also notice a double asterisk (**) at select points in the text. These ** designate important concepts outside the definitions that will be essential in the reasoning and problem solving components of Geometry.

In the Questions portion of each section, there are four types of questions and are designated to help you understand the level of the required knowledge to answer each one. Question with a Ⓔ designation indicate an "essential" question and is part of the foundational material for the course. No designation indicates that the question will involve "problem solving" techniques and most likely will require knowledge of multiple concepts and require multiple steps to solve. As the course progresses, you will notice the increase of problem solving in some of the essential questions. The **Challenge** questions will require higher level complex problem solving techniques and the concepts needed to solve will span a larger number of topics. The "Advanced Challenge" questions will be quite complex and may require interdisciplinary knowledge.

Note cards

While having the notes and an ability to mark them up is nice and allows you to focus on what is being discussed in class rather than madly taking notes, it also takes that important learning (and retention) tool away from you. To compensate for this and to make sure that you absorb the concepts and relationships of this course, the first question in almost every section asks you to make note cards for the primary vocabulary and geometric relationships (essential information). It is important to take your time with these note cards. Note cards should include the term on the front of the card. The back of the card should include:

1. A few words of definition/description (it does not need to be word for word from the text, it should be something that makes sense to you).
2. An example or diagram of its use – in Geometry this is probably the most important part.
3. Any notation/symbols or relationships (ex: perpendicular = $\perp$)
4. Write down any properties that may be outside the definition, but give a fuller understanding of the term.
5. Write down any relationships to other topics or terms

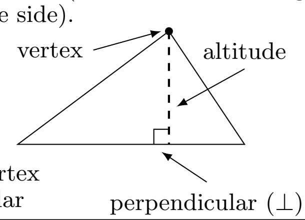
Here is an example of a complete note card (front and back):

Front of Card

Altitude
(of a triangle)

Back of Card

An altitude of a traingle is a segment from one vertex, drawn perpendicular to the opposite side (or the line containing the opposite side).



Related
triangle, vertex
perpendicular

What is Geometry?

Geometry is the field of mathematics concerned with shapes, including their sizes, positions, and relationships. One major area of focus will be on finding lengths, areas, and volumes. We will also delve into the ideas of congruence and similarity, that is, whether two shapes are identical copies, or whether one is an enlargement of the other. We will spend most of our time studying triangles, quadrilaterals, and circles.

There will also be a major focus on mathematical proof this year. Mathematical proof is the method by which you reason through a problem and write your answer in a clear, logical fashion.

We are going to use geometry as a backdrop for studying mathematical proof in the same way that your art teacher might use a bowl of fruit as a subject for how to understand shadow and shape. It's not about the fruit, it's about the painting you make from it.

You probably already learned the Pythagorean Theorem (the most important result in this course, hands down). In this course, it is not enough to *know* the Pythagorean Theorem, you must be able to explain *why* it is true.

Get used to being asked *Why?*. That is what geometry will be about.

Geometry vs. Algebra

When you took algebra, most of your answers were probably numbers. You would get an answer like $x = 7$, and you would put a box around it and go on your merry way.

$$\boxed{\text{x=7}} \quad \ddot\smile$$

Geometry answers, in contrast, are frequently sentences or paragraphs.

If you want to be in a position to do well, pay attention to the *vocabulary* and the *notation* in the early going. Vocabulary refers to the words that are used, and notation refers to the proper way of writing them. You won't see as many equations as you did last year, but rest assured, this is still very much a math class. In fact, some may consider it to be more of a "math" class!

Many people find the beginning of geometry very easy because we will spend so much time learning basic vocabulary and doing simple drawings. Don't become complacent. If you do not learn the early language, you will be in trouble later. You should know that the course will gradually increase in difficulty.

More than other math classes, Geometry builds upon itself. The level at which you learn to reason and problem-solve in the first few months will dictate how hard or how easy this course is in the second semester. Those of you that understand the basics well should see success through the end of the year.

Euclidean Geometry

There is Arena Football, Australian Rules Football, International Football (that we call Soccer), and then of course, there is the NFL. Like football, geometry has a few different varieties as well. We will study Euclidean Geometry which is the oldest and most intuitive form of geometry, and the one that everybody else studies too. It was first written down by Euclid, more than two thousand years ago. In the first semester, it can be assumed that everything is coplanar (that is, in the same plane: on a piece of paper, on the whiteboard, or on the Smartboard) unless explicitly stated otherwise. In the latter portion of the year, we will examine solids (3-D Objects). Anything that has 3-Dimensions is not coplanar.

Course Assumptions

In mathematics, it is generally not a good idea to assume something is the case if you have not been told so or you can not prove it. In a class like Geometry, we establish some course assumptions to make our writing more concise or make it easier to interpret drawings and sketches. For example:

We *will* assume that if a point appears on a line then it is on that line.

We *will not* assume that if two lines look parallel then they are parallel.

These course assumptions will be clearly stated throughout the text.

Algebra review

It may be necessary for you to review some of the algebra that we will be using regularly. The pointers below are intended to refresh your memory of things you already know. If an item looks completely unfamiliar, let me know.

Solving a linear equation

Linear equations do not have an x^2 term. You typically aim to isolate the variable you are solving for by performing the same operation on both sides (algebraic properties of equality).

Solving a quadratic equation

Quadratic equations have an x^2 term. Quadratic equations must first be set to zero. Then you may either (a) factor, or (b) use the quadratic formula $\left[x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}\right]$

Solving a proportion

Proportions are most easily solved by "cross-multiplying", and solving the resulting equation, whether it be linear or quadratic.

Solving a system

A system of equations is a group of two equations with two unknowns. There are many methods, but the most popular are (a) substitution or (b) linear elimination.

With substitution, you solve one of the equations for one of the variables, and then substitute that quantity into the other equation.

With linear elimination, you multiply each equation by some constant so that a variable will cancel when the equations are added.

Graphing lines

You probably remember that knowing two points is enough to write the equation of a line.

The easiest way is typically to find the slope of the line $\left(m = \frac{y_2-y_1}{x_2-x_1}\right)$, and then to use point-slope form $y - y_1 = m(x - x_1)$ or slope-intercept form $y = mx + b$.

Parallel lines have equal slopes, and perpendicular lines have slopes that are opposite reciprocals.

Exercises

Question 0.0.1. In the following equations, solve for the variable:

(a) $3x + 2 = 14$

(b) $3(a + 2) = 15$

(c) $4k + 5 = k - 6$

(d) $-4(x + 2) = 5x - 17$

(e) $x^2 = 25$

(f) $m^2 + m - 12 = 0$

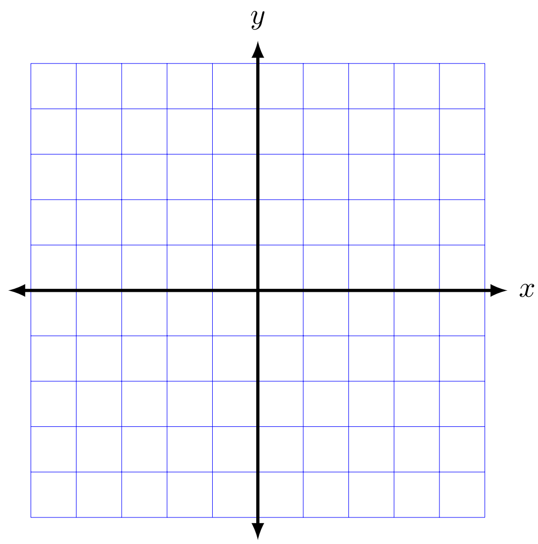
(g) $s^2 = 4s + 12$

(h) **Challenge** $2y^2 - 11y + 14 = 0$

(i) $\dfrac{3}{x - 2} = \dfrac{5}{3x + 4}$

Question 0.0.2. On the axes provided below, plot and label (with coordinates) points $A(-1,-4)$ and $B(3,4)$.

(a) Sketch the line that passes through A and B.



(b) What is the slope of the line that passes through these points?

(c) Write the equation of the line that passes through A and B.

(d) What is the slope of any line that is parallel to the line passing through A and B?

(e) Write the equation of a line parallel to the original line that passes through the point $(-2,2)$.

(f) What is the slope of the line perpendicular to the original line?

(g) Write the equation of a line perpendicular to the original line that passes through the point $(4,2)$.

(h) What are the x and y intercepts of the perpendicular line in (g)?

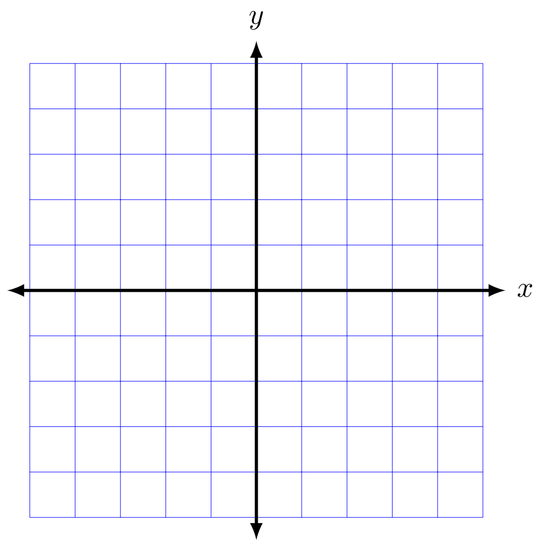
(i) **Challenge** Give the point of intersection of the original line through A and B and the perpendicular line in (g).

Question 0.0.3. Solve the system of equations for x and y:

$$\begin{aligned} 3x - y &= -2 \\ 5x - y &= 4 \end{aligned}$$

Question 0.0.4. On the axes provided below, plot and label (with coordinates) points $E(1,2)$ and $F(5,2)$.

(a) Sketch the line that passes through E and F.



(b) What is the slope of the line that passes through these points?

(c) Write the equation of the line that passes through E and F.

(d) Give the x and y intercepts of the line in part (c).

(e) Graph the line $x = -3$.

(f) Give the slope of the line in part (e).

(g) Give the x and y intercepts of the line in part (e).

(h) Give the point of intersection of the two lines in parts (c) and (e).

Question 0.0.5. How far does a car travel if it is moving at 80 mi/h for 3 hours?

Question 0.0.6. In the following equations, solve for the variable:

(a) $2 = 4z - 18$

(b) $4(b + 3) = 18$

(c) $7k - 4 = 2k + 11$

(d) $-3(x + 2) + 5x = 4$

(e) $x^2 = 81$

(f) $p^2 + p - 20 = 0$

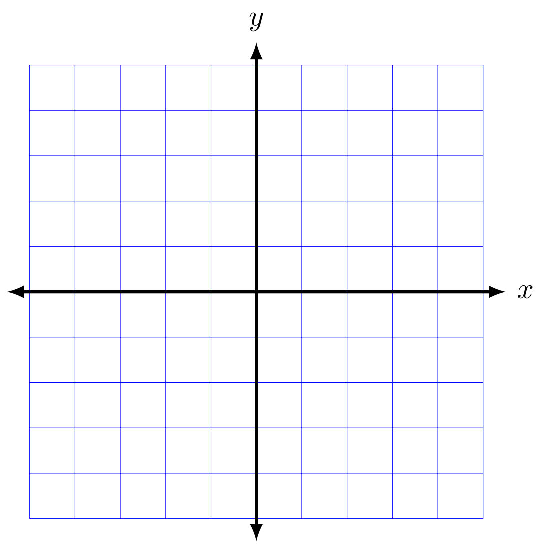
(g) $s^2 = 7s + 18$

(h) $\dfrac{3b}{4} = \dfrac{4b + 3}{6}$

(i) **Challenge** $\dfrac{2z - 5}{z^2 + 1} = \dfrac{1}{z + 1}$

Question 0.0.7. On the axes provided below, plot the line: $2x + 3y = 9$.

(a) Label the coordinates of the x and y intercepts.



(b) Give the equation of the line in slope-intercept form ($y = mx + b$).

(c) What is the slope of the line?

(d) Write the equation of the line parallel to the original line that passes through the origin.

(e) Write the equation of a line perpendicular to the original line that passes through the point $(1, -2)$.

(f) **Challenge** Give the point of intersection of the original line and the perpendicular line in (e).

Question 0.0.8. Solve the system of equations for a and b:

$$\begin{aligned} 4a + 2b &= 5 \\ a + 3b &= 5 \end{aligned}$$

Question 0.0.9. Challenge: If a commuter train leaves 30th Street Station at noon traveling at 30 m/s and another high-speed train leaves the station 30 minutes later traveling at 50 m/s. At what time does the second train catch up to the first train? Make a drawing of the problem and define the variables. Solve the problem using your defined variables.

Question 0.0.10. Challenge: In the following equations, solve for x:

(a) $ax + bx = 4$

(b) $x^2 - a = 0$

Answers

Appendix C

Answers to exercises

Section 0.0 Solutions

0.0.1 (a) 4

(b) 3

(c) $-\frac{11}{3}$

(d) 1

(e) ± 5

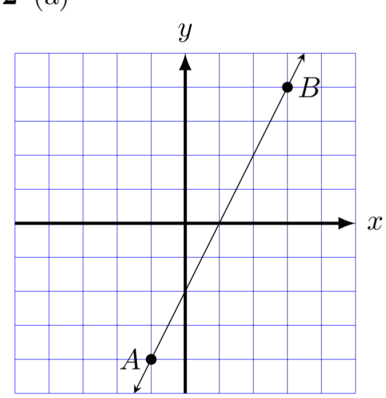
(f) 3 or -4

(g) 6 or -2

(h) 2 or $\frac{7}{2}$

(i) $-\frac{11}{2}$

0.0.2 (a)



(b) 2

(c) $y = 2x - 2$

(d) 2

(e) $y = 2x + 6$

(f) $-\frac{1}{2}$

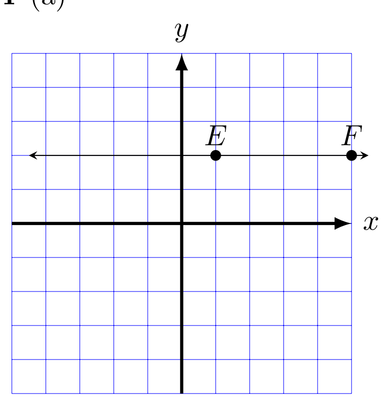
(g) $y = -\frac{1}{2}x + 4$

(h) $(8, 0)$ and $(0, 4)$

(i) $(\frac{12}{5}, \frac{14}{5})$

0.0.3 $(3, 11)$

0.0.4 (a)



(b) 0

(c) $y = 2$

(d) y-int @ $(0, 2)$, no x-int

(e) vertical line through (-3,0)

(f) undefined

(g) x-int @ $(-3, 0)$, no y-int

(h) $(-3, 2)$

0.0.5 240 miles

0.0.6 (a) 5

(b) $\frac{3}{2}$

(c) 3

(d) 5

(e) ± 9

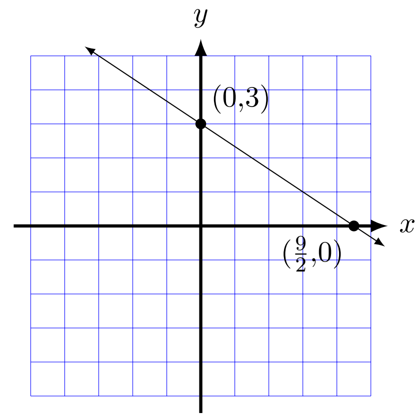
(f) 4 or -5

(g) 9 or -2

(h) 6

(i) $\frac{3 \pm \sqrt{33}}{2}$

0.0.7 (a)



(b) $y = -\frac{2}{3}x + 3$

(c) $-\frac{2}{3}$

(d) $y = -\frac{2}{3}x$

(e) $y = \frac{3}{2}x - \frac{7}{2}$

(f) (3,1)

0.0.8 $(a, b) = (\frac{1}{2}, \frac{3}{2})$

0.0.9 1:15 pm

0.0.10 (a) $\frac{4}{a+b}$

(b) $\pm\sqrt{a}$

Back to questions