1997 AP Calculus AB: Section I, Part A

50 Minutes—No Calculator

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_1^2 (4x^3 - 6x)\,dx =$

(A) 2
(B) 4
(C) 6
(D) 36
(E) 42

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2. If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

(A) $\dfrac{3x-3}{\sqrt{2x-3}}$

(B) $\dfrac{x}{\sqrt{2x-3}}$

(C) $\dfrac{1}{\sqrt{2x-3}}$

(D) $\dfrac{-x+3}{\sqrt{2x-3}}$

(E) $\dfrac{5x-6}{2\sqrt{2x-3}}$

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3. If $\int_a^b f(x)\,dx = a + 2b$, then $\int_a^b (f(x)+5)\,dx =$

(A) $a+2b+5$ (B) $5b-5a$ (C) $7b-4a$ (D) $7b-5a$ (E) $7b-6a$

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4. If $f(x) = -x^3 + x + \dfrac{1}{x}$, then $f'(-1) =$

(A) 3 (B) 1 (C) -1 (D) -3 (E) -5

5. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

(A) $x < 0$

(B) $x > 0$

(C) $x < -2$ or $x > -\frac{2}{3}$

(D) $x < \frac{2}{3}$ or $x > 2$

(E) $\frac{2}{3} < x < 2$

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6. $\frac{1}{2}\int e^{\frac{t}{2}}\,dt =$

(A) $e^{-t} + C$ (B) $e^{-\frac{t}{2}} + C$ (C) $e^{\frac{t}{2}} + C$ (D) $2e^{\frac{t}{2}} + C$ (E) $e^{t} + C$

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7. $\frac{d}{dx}\cos^2(x^3) =$

(A) $6x^2 \sin(x^3)\cos(x^3)$

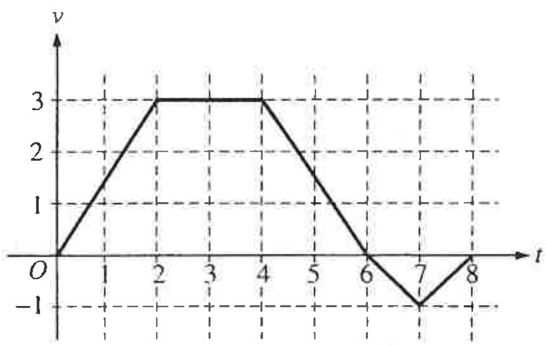
(B) $6x^2 \cos(x^3)$

(C) $\sin^2(x^3)$

(D) $-6x^2 \sin(x^3)\cos(x^3)$

(E) $-2\sin(x^3)\cos(x^3)$

Questions 8-9 refer to the following situation.



A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

8. At what value of t does the bug change direction?

(A) 2 (B) 4 (C) 6 (D) 7 (E) 8

9. What is the total distance the bug traveled from $t = 0$ to $t = 8$?

(A) 14 (B) 13 (C) 11 (D) 8 (E) 6

10. An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$ is

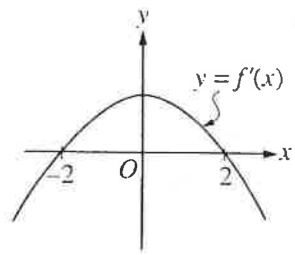
(A) $y - 1 = -\left(x - \frac{\pi}{4}\right)$

(B) $y - 1 = -2\left(x - \frac{\pi}{4}\right)$

(C) $y = 2\left(x - \frac{\pi}{4}\right)$

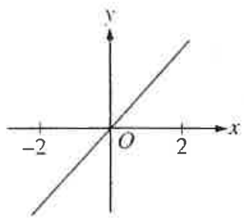
(D) $y = -\left(x - \frac{\pi}{4}\right)$

(E) $y = -2\left(x - \frac{\pi}{4}\right)$

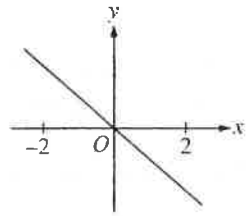


11. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

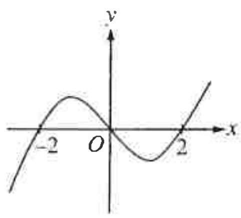
(A)



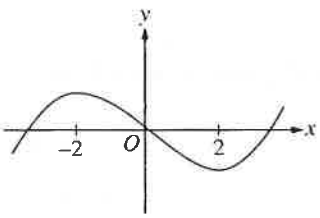
(B)



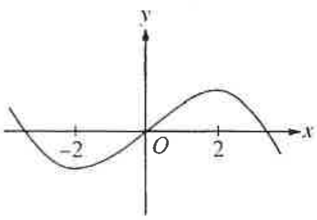
(C)



(D)



(E)



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12. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

(A) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (B) $\left(\frac{1}{2}, \frac{1}{8}\right)$ (C) $\left(1, -\frac{1}{4}\right)$ (D) $\left(1, \frac{1}{2}\right)$ (E) $(2, 2)$

13. Let f be a function defined for all real numbers x. If $f'(x) = \dfrac{|4-x^2|}{x-2}$, then f is decreasing on the interval

(A) $(-\infty, 2)$ (B) $(-\infty, \infty)$ (C) $(-2, 4)$ (D) $(-2, \infty)$ (E) $(2, \infty)$

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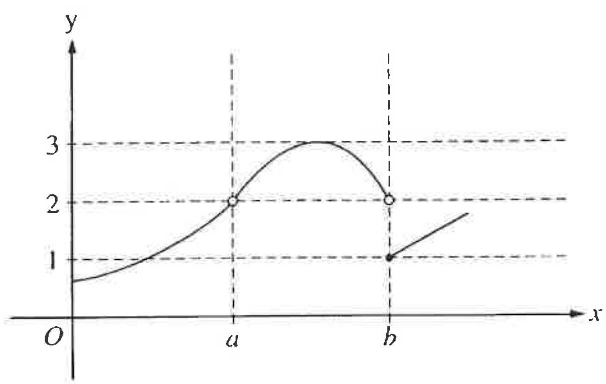
14. Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $x = 3$ is used to find an approximation to a zero of f, that approximation is

(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5



15. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$

(B) $\lim_{x \to a} f(x) = 2$

(C) $\lim_{x \to b} f(x) = 2$

(D) $\lim_{x \to b} f(x) = 1$

(E) $\lim_{x \to a} f(x)$ does not exist.

16. The area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$ is

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{28}{3}$ (D) $\frac{32}{3}$ (E) 8π

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17. If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4,3)$?

(A) $-\frac{25}{27}$ (B) $-\frac{7}{27}$ (C) $\frac{7}{27}$ (D) $\frac{3}{4}$ (E) $\frac{25}{27}$

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18. $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}\,dx$ is

(A) 0 (B) 1 (C) $e-1$ (D) e (E) $e+1$

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19. If $f(x) = \ln\left|x^2 - 1\right|$, then $f'(x) =$

(A) $\left|\frac{2x}{x^2-1}\right|$

(B) $\frac{2x}{\left|x^2-1\right|}$

(C) $\frac{2|x|}{x^2-1}$

(D) $\frac{2x}{x^2-1}$

(E) $\frac{1}{x^2-1}$

20. The average value of $\cos x$ on the interval $[-3,5]$ is

(A) $\dfrac{\sin 5-\sin 3}{8}$

(B) $\dfrac{\sin 5-\sin 3}{2}$

(C) $\dfrac{\sin 3-\sin 5}{2}$

(D) $\dfrac{\sin 3+\sin 5}{2}$

(E) $\dfrac{\sin 3+\sin 5}{8}$

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21. $\lim\limits_{x\to 1}\dfrac{x}{\ln x}$ is

(A) 0 (B) $\dfrac{1}{e}$ (C) 1 (D) e (E) nonexistent

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22. What are all values of x for which the function f defined by $f(x)=(x^2-3)e^{-x}$ is increasing?

(A) There are no such values of x.

(B) $x<-1$ and $x>3$

(C) $-3<x<1$

(D) $-1<x<3$

(E) All values of x

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23. If the region enclosed by the y-axis, the line $y=2$, and the curve $y=\sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

(A) $\dfrac{32\pi}{5}$ (B) $\dfrac{16\pi}{3}$ (C) $\dfrac{16\pi}{5}$ (D) $\dfrac{8\pi}{3}$ (E) π

24. The expression $\frac{1}{50}\left(\sqrt{\frac{1}{50}}+\sqrt{\frac{2}{50}}+\sqrt{\frac{3}{50}}+\cdots+\sqrt{\frac{50}{50}}\right)$ is a Riemann sum approximation for

(A) $\int_0^1 \sqrt{\frac{x}{50}}\,dx$

(B) $\int_0^1 \sqrt{x}\,dx$

(C) $\frac{1}{50}\int_0^1 \sqrt{\frac{x}{50}}\,dx$

(D) $\frac{1}{50}\int_0^1 \sqrt{x}\,dx$

(E) $\frac{1}{50}\int_0^{50} \sqrt{x}\,dx$

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25. $\int x\sin(2x)\,dx =$

(A) $-\frac{x}{2}\cos(2x)+\frac{1}{4}\sin(2x)+C$

(B) $-\frac{x}{2}\cos(2x)-\frac{1}{4}\sin(2x)+C$

(C) $\frac{x}{2}\cos(2x)-\frac{1}{4}\sin(2x)+C$

(D) $-2x\cos(2x)+\sin(2x)+C$

(E) $-2x\cos(2x)-4\sin(2x)+C$

1997 AP Calculus AB: Section I, Part B

40 Minutes—Graphing Calculator Required

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

76. If $f(x) = \dfrac{e^{2x}}{2x}$, then $f'(x) =$

(A) 1

(B) $\dfrac{e^{2x}(1-2x)}{2x^2}$

(C) e^{2x}

(D) $\dfrac{e^{2x}(2x+1)}{x^2}$

(E) $\dfrac{e^{2x}(2x-1)}{2x^2}$

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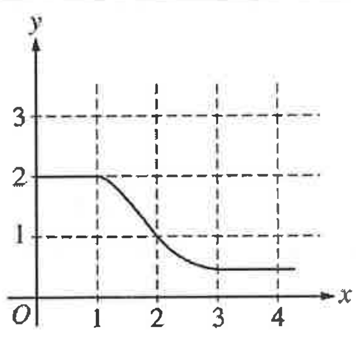
77. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$

(A) -1.58 (B) -1.63 (C) -1.67 (D) -1.89 (E) -2.33



78. The graph of f is shown in the figure above. If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

(A) 0.3 (B) 1.3 (C) 3.3 (D) 4.3 (E) 5.3

79. Let f be a function such that $\lim_{h \to 0} \frac{f(2+h)-f(2)}{h} = 5$. Which of the following must be true?

I. f is continuous at $x = 2$.

II. f is differentiable at $x = 2$.

III. The derivative of f is continuous at $x = 2$.

(A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

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80. Let f be the function given by $f(x) = 2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 3?

(A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

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81. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

(A) 57.60 (B) 57.88 (C) 59.20 (D) 60.00 (E) 67.40

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82. If $y = 2x - 8$, what is the minimum value of the product xy?

(A) -16 (B) -8 (C) -4 (D) 0 (E) 2

---

83. What is the area of the region in the first quadrant enclosed by the graphs of $y = \cos x$, $y = x$, and the y-axis?

(A) 0.127 (B) 0.385 (C) 0.400 (D) 0.600 (E) 0.947

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84. The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln x}$, the line $x = e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 2 (E) $\frac{1}{3}(e^3 - 1)$

85. If the derivative of f is given by $f'(x) = e^x - 3x^2$, at which of the following values of x does f have a relative maximum value?

(A) −0.46 (B) 0.20 (C) 0.91 (D) 0.95 (E) 3.73

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86. Let $f(x) = \sqrt{x}$. If the rate of change of f at $x = c$ is twice its rate of change at $x = 1$, then $c =$

(A) $\frac{1}{4}$ (B) 1 (C) 4 (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{1}{2\sqrt{2}}$

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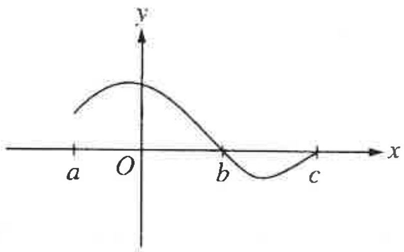
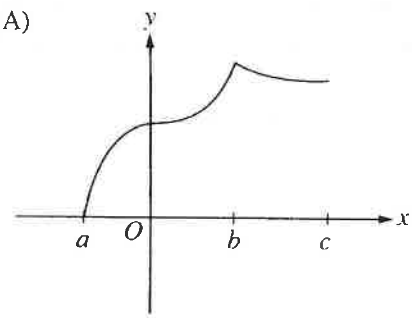
87. At time $t \geq 0$, the acceleration of a particle moving on the x-axis is $a(t) = t + \sin t$. At $t = 0$, the velocity of the particle is −2. For what value t will the velocity of the particle be zero?

(A) 1.02 (B) 1.48 (C) 1.85 (D) 2.81 (E) 3.14

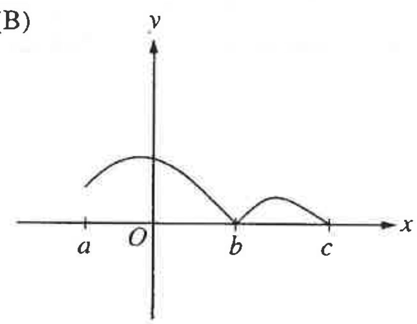


88. Let $f(x)=\int_a^x h(t)\,dt$, where h has the graph shown above. Which of the following could be the graph of f?

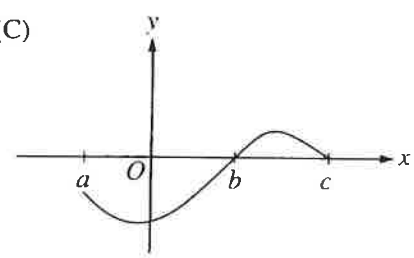
(A)



(B)



(C)



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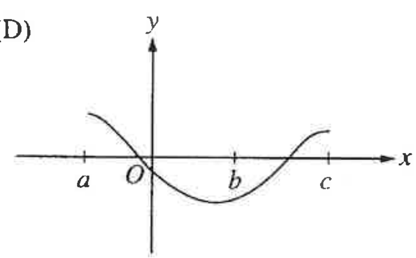
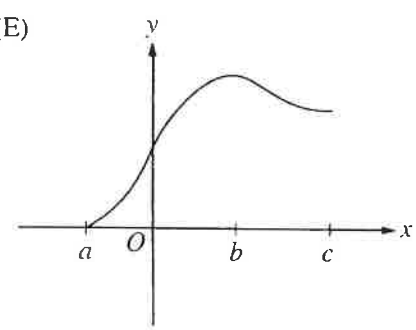


(E)



x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

89. A table of values for a continuous function f is shown above. If four equal subintervals of $[0,2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

(A) 8 (B) 12 (C) 16 (D) 24 (E) 32

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90. Which of the following are antiderivatives of $f(x) = \sin x \cos x$?

I. $F(x) = \dfrac{\sin^2 x}{2}$

II. $F(x) = \dfrac{\cos^2 x}{2}$

III. $F(x) = \dfrac{-\cos(2x)}{4}$

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only

1997 Calculus AB Solutions: Part A

1. C $\int_1^2 (4x^3 - 6x)\,dx = (x^4 - 3x^2)\Big|_1^2 = (16-12)-(1-3) = 6$

2. A $f(x) = x(2x-3)^{\frac{1}{2}};\ f'(x) = (2x-3)^{\frac{1}{2}} + x(2x-3)^{-\frac{1}{2}} = (2x-3)^{-\frac{1}{2}}(3x-3) = \dfrac{(3x-3)}{\sqrt{2x-3}}$

3. C $\int_a^b (f(x)+5)\,dx = \int_a^b f(x)dx + 5\int_a^b 1\,dx = a + 2b + 5(b-a) = 7b - 4a$

4. D $f(x) = -x^3 + x + \dfrac{1}{x};\ f'(x) = -3x^2 + 1 - \dfrac{1}{x^2};\ f'(-1) = -3(-1)^2 + 1 - \dfrac{1}{(-1)^2} = -3+1-1 = -3$

5. E $y = 3x^4 - 16x^3 + 24x^2 + 48;\ y' = 12x^3 - 48x^2 + 48x;\ y'' = 36x^2 - 96x + 48 = 12(3x-2)(x-2)$
$y'' < 0$ for $\dfrac{2}{3} < x < 2$, therefore the graph is concave down for $\dfrac{2}{3} < x < 2$

6. C $\dfrac{1}{2}\int e^{\frac{t}{2}}\,dt = e^{\frac{t}{2}} + C$

7. D $\dfrac{d}{dx}\cos^2(x^3) = 2\cos(x^3)\left(\dfrac{d}{dx}(\cos(x^3)\right) = 2\cos(x^3)(-\sin(x^3)\left(\dfrac{d}{dx}(x^3)\right)$
$= 2\cos(x^3)(-\sin(x^3)(3x^2)$

8. C The bug change direction when v changes sign. This happens at $t = 6$.

9. B Let A_1 be the area between the graph and t-axis for $0 \le t \le 6$, and let A_2 be the area between the graph and the t-axis for $6 \le t \le 8$ Then $A_1 = 12$ and $A_2 = 1$. The total distance is $A_1 + A_2 = 13$.

10. E $y = \cos(2x);\ y' = -2\sin(2x);\ y'\left(\dfrac{\pi}{4}\right) = -2$ and $y\left(\dfrac{\pi}{4}\right) = 0;\ y = -2\left(x - \dfrac{\pi}{4}\right)$

11. E Since f' is positive for $-2 < x < 2$ and negative for $x < -2$ and for $x > 2$, we are looking for a graph that is increasing for $-2 < x < 2$ and decreasing otherwise. Only option E.

12. B $y = \dfrac{1}{2}x^2;\ y' = x;$ We want $y' = \dfrac{1}{2} \Rightarrow x = \dfrac{1}{2}$. So the point is $\left(\dfrac{1}{2}, \dfrac{1}{8}\right)$.

13. A $f'(x)=\frac{|4-x^2|}{x-2}$; f is decreasing when $f'<0$. Since the numerator is non-negative, this is only when the denominator is negative. Only when $x<2$.

14. C $f(x)\approx L(x)=2+5(x-3)$; $L(x)=0$ if $0=5x-13 \Rightarrow x=2.6$

15. B Statement B is true because $\lim_{x\to a^-} f(x)=2=\lim_{x\to a^+} f(x)$. Also, $\lim_{x\to b} f(x)$ does not exist because the left- and right-sided limits are not equal, so neither (A), (C), nor (D) are true.

16. D The area of the region is given by $\int_{-2}^{2}(5-(x^2+1))dx=2(4x-\frac{1}{3}x^3)\Big|_0^2=2\left(8-\frac{8}{3}\right)=\frac{32}{3}$

17. A $x^2+y^2=25$; $2x+2y\cdot y'=0$; $x+y\cdot y'=0$; $y'(4,3)=-\frac{4}{3}$;

$x+y\cdot y'=0 \Rightarrow 1+y\cdot y''+y'\cdot y'=0$; $1+(3)y''+\left(-\frac{4}{3}\right)\cdot\left(-\frac{4}{3}\right)=0$; $y''=-\frac{25}{27}$

18. C $\int_0^{\frac{\pi}{4}}\frac{e^{\tan x}}{\cos^2 x}dx$ is of the form $\int e^u du$ where $u=\tan x$..

$\int_0^{\frac{\pi}{4}}\frac{e^{\tan x}}{\cos^2 x}dx=e^{\tan x}\Big|_0^{\frac{\pi}{4}}=e^1-e^0=e-1$

19. D $f(x)=\ln|x^2-1|$; $f'(x)=\frac{1}{x^2-1}\cdot\frac{d}{dx}(x^2-1)=\frac{2x}{x^2-1}$

20. E $\frac{1}{8}\int_{-3}^{5}\cos x\,dx=\frac{1}{8}(\sin 5-\sin(-3))=\frac{1}{8}(\sin 5+\sin 3)$; Note: Since the sine is an odd function, $\sin(-3)=-\sin(3)$.

21. E $\lim_{x\to 1}\frac{x}{\ln x}$ is nonexistent since $\lim_{x\to 1}\ln x=0$ and $\lim_{x\to 1} x\neq 0$.

22. D $f(x)=(x^2-3)e^{-x}$; $f'(x)=e^{-x}(-x^2+2x+3)=-e^{-x}(x-3)(x+1)$; $f'(x)>0$ for $-1<x<3$

23. A Disks where $r=x$. $V=\pi\int_0^2 x^2dy=\pi\int_0^2 y^4dy=\frac{\pi}{5}y^5\Big|_0^2=\frac{32\pi}{5}$

24. B Let $[0,1]$ be divided into 50 subintervals. $\Delta x = \frac{1}{50}$; $x_1 = \frac{1}{50}, x_2 = \frac{2}{50}, x_3 = \frac{3}{50}, \cdots, x_{50} = 1$

Using $f(x) = \sqrt{x}$, the right Riemann sum $\sum_{i=1}^{50} f(x_i)\Delta x$ is an approximation for $\int_0^1 \sqrt{x}\, dx$.

25. A Use the technique of antiderivatives by parts, which was removed from the AB Course Description in 1998.

$$u = x \qquad dv = \sin 2x\, dx$$

$$du = dx \qquad v = -\frac{1}{2}\cos 2x$$

$$\int x\sin(2x)\, dx = -\frac{1}{2}x\cos(2x) + \int \frac{1}{2}\cos(2x)\, dx = -\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) + C$$

76. E $f(x)=\dfrac{e^{2x}}{2x}$; $f'(x)=\dfrac{2e^{2x}\cdot 2x-2e^{2x}}{4x^2}=\dfrac{e^{2x}(2x-1)}{2x^2}$

77. D $y=x^3+6x^2+7x-2\cos x$. Look at the graph of $y''=6x+12+2\cos x$ in the window [–3,–1] since that domain contains all the option values. y'' changes sign at $x=-1.89$.

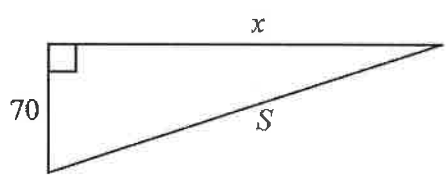
78. D $F(3)-F(0)=\int_0^3 f(x)dx=\int_0^1 f(x)dx+\int_1^3 f(x)dx=2+2.3=4.3$

(Count squares for $\int_0^1 f(x)dx$)

79. C The stem of the questions means $f'(2)=5$. Thus f is differentiable at $x=2$ and therefore continuous at $x=2$. We know nothing of the continuity of f'. I and II only.

80. A $f(x)=2e^{4x^2}$; $f'(x)=16xe^{4x^2}$; We want $16xe^{4x^2}=3$. Graph the derivative function and the function $y=3$, then find the intersection to get $x=0.168$.

81. A Let x be the distance of the train from the crossing. Then $\dfrac{dx}{dt}=60$.



$S^2=x^2+70^2\Rightarrow 2S\dfrac{dS}{dt}=2x\dfrac{dx}{dt}\Rightarrow\dfrac{dS}{dt}=\dfrac{x}{S}\dfrac{dx}{dt}$.

After 4 seconds, $x=240$ and so $S=250$.

Therefore $\dfrac{dS}{dt}=\dfrac{240}{250}(60)=57.6$

82. B $P(x)=2x^2-8x$; $P'(x)=4x-8$; P' changes from negative to positive at $x=2$. $P(2)=-8$

83. C $\cos x=x$ at $x=0.739085$. Store this in A. $\int_0^A(\cos x-x)\,dx=0.400$

84. C Cross sections are squares with sides of length y.

Volume $=\int_1^e y^2dx=\int_1^e \ln x\,dx=(x\ln x-x)\Big|_1^e=(e\ln e-e)-(0-1)=1$

85. C Look at the graph of f' and locate where the y changes from positive to negative. $x=0.91$

86. A $f(x)=\sqrt{x}$; $f'(x)=\dfrac{1}{2\sqrt{x}}$; $\dfrac{1}{2\sqrt{c}}=2\cdot\dfrac{1}{2\sqrt{1}}\Rightarrow c=\dfrac{1}{4}$

87. B $a(t) = t + \sin t$ and $v(0) = -2 \Rightarrow v(t) = \frac{1}{2}t^2 - \cos t - 1$; $v(t) = 0$ at $t = 1.48$

88. E $f(x) = \int_a^x h(x)dx \Rightarrow f(a) = 0$, therefore only (A) or (E) are possible. But $f'(x) = h(x)$ and therefore f is differentiable at $x = b$. This is true for the graph in option (E) but not in option (A) where there appears to be a corner in the graph at $x = b$. Also, Since h is increasing at first, the graph of f must start out concave up. This is also true in (E) but not (A).

89. B $T = \frac{1}{2} \cdot \frac{1}{2}(3 + 2 \cdot 3 + 2 \cdot 5 + 2 \cdot 8 + 13) = 12$

90. D

$F(x) = \frac{1}{2}\sin^2 x$	$F'(x) = \sin x \cos x$	Yes
$F(x) = \frac{1}{2}\cos^2 x$	$F'(x) = -\cos x \sin x$	No
$F(x) = -\frac{1}{4}\cos(2x)$	$F'(x) = \frac{1}{2}\sin(2x) = \sin x \cos x$	Yes