Upper School Mathematics FAQ

Preparing Boys *for Life.*

Here, we address a few of the most commonly asked questions in math education. We hope it will prompt you to speak with your son or his teachers to learn more about the scope of the mathematics curriculum at The Haverford School.

Q: My son does well on his math homework and quizzes, but tends not to do as well on the comprehensive tests. Why is that?

A: The answer to this question can likely be applied to learning in general, not just mathematics. It is not unusual for a student to earn a grade lower on a cumulative test or semester exam than he has been averaging on his homework or quizzes. A unit test requires the assimilation of many skills and ideas, whereas a quiz or homework assignment will focus on one or two topics. While your son may have mastered one or two skills to earn an "A" on a quiz, he will be required to recognize *which skill* is needed when he is confronted with a cumulative test. Unless he has grappled with this type of preparation before the assessment, he is likely to feel confused by the synthesis of ideas. Additionally, by the end of a unit his teacher will assume that he can apply his newly learned skills to solve problems he has not yet seen. His teacher has equipped him with a fully stocked toolbox to complete the test, but he will need to determine which tool to use to get the job done.

In *The New Science of Learning*, authors Terry Doyle and Todd Zakrajsek tell us that psychologists refer to this as transference. "It is easy to memorize information and then transfer it to an identical scenario. Your instructor is trying to help you understand the math by seeing whether you can use the knowledge you have developed to solve new problems." [1]

What can your son do to improve his cumulative assessment scores? He should practice all the skills he has learned in a unit for a short time each day. Typically his math teacher will assign a few problems from previous exercises that serve as a review, but if not, he should do this himself. Doyle reminds us that "Even when you have learned something, if you don't practice what you have learned, the information fades." [2] A second suggestion is to simulate a comparable test situation. This involves choosing a selection of problems from the unit that vary in difficulty and working them within the same time frame as the actual test. This will give him practice searching his toolbox as well as identifying which tools need to be honed.

Q: What are the biggest differences in math education from when I was studying in middle and high school?

A: Depending on when you matriculated, the differences could be quite vast. When I was in high school, mathematics was taught in finite units that appeared to have little connection to one another, let alone relevance to other subjects. It was highly skill-based, which typically appeals to two types of students: rule followers and those who are more mathematically inclined. Today, math educators recognize that teaching a skill and then assessing that skill is not likely to create problem solvers. Universities and employers certainly want individuals capable of learning new skills, but more importantly they want individuals who can *use* their knowledge to devise solutions to problems that don't yet exist! Your son may be repeatedly told to "show his work" or that the final answer is not worth as many points as the steps leading to the final answer. This is because mathematics education is more about the *process of finding the answer* than the actual answer itself. Mathematics teaches reasoning, logic, justification, and critical thinking. It may surprise you to know that undergraduate students taking the LSAT from majors like math, physics and economics consistently rank among the highest scorers. These areas all require *application*, not *regurgitation* of knowledge.

As a parent, you may find that even your Lower School son is grappling with how to complete his homework. He may even say things like, "We have never done anything like this in class!" That may in fact be true, but you can remind him that although he has never been confronted with a particular problem, his teacher has supplied him with all the necessary tools. It is up to him to find the best one for the job.

Q: What are the primary differences between standard level math courses and Honors math courses?

A: In Upper School, the math department offers two levels of most math courses. Although we strongly believe that all boys are capable of successfully learning high-level mathematics, we understand that interests vary, and our students must allocate their time to the topics they are most interested in or believe they will study at college. Our goal is to prepare them to successfully transition into the next level of their math education. The standard level courses offer all the skills and ideas deemed to fit under a particular topic like Geometry or Algebra II. However, your son's teacher may allocate more time to mastering skills, offer more guidance on problem solving, or require less independent exploration. In the Honors courses, the boys are expected to master skills quickly, have a well developed problem solving process, read and interpret mathematics independently, and readily move on to more demanding applications of the material.

A daily commitment to practice is required for both levels, but in an Honors section where new topics are introduced daily, it is imperative. The teachers of our Honors sections may require the boys to discover content instead of being taught by the teacher. This requires energy and perseverance. For a boy who is less interested in mathematics, he may not be as diligent and will quickly fall behind his Honors classmates.

Regardless of one's math level, achieving success in mathematics requires daily practice. Period. It requires engagement, communication, trial and error, and lots of trips to the toolbox. The rewards of this type of approach transcend mathematics, which Doyle captures when he says, "What neuroscience researchers have made clear is that the one who does the work does the learning." [3] When trying to create long-term memories from new information, "there is no substitute for practice." [4]

Q: Why all the fuss about algebra readiness?

A: You're right, we do make a big deal about algebra readiness, and this is not just the case at The Haverford School. I frequently refer parents to an article entitled "*Algebra: Not if, but When,*" written by Linda Gojak, former president of the National Council of Teachers of Mathematics.[5] I find it very informative on the topic of student readiness for algebra. I would encourage you to read it yourself, but will summarize a few of the main points.

In the article, Gojak describes algebra as the gatekeeper to higher learning, and you may have heard your son's teacher say the same, because we *really* believe this! Whenever I hear the term gatekeeper, my mind immediately conjures up a vision of Gandalf facing the Balrog on the narrow bridge of the abyss in "*Lord of the Rings,*" commanding: "You shall not pass!" Well, math teachers are a bit like this when it comes to algebra. However, we would add the word "yet;" Gandalf had the big picture in mind when he let go of the ledge, and your son's teacher knows your son will be stronger in the end if this foundational course is well aligned with his developmental readiness.

Algebra depends on one's ability to think abstractly, meaning the ability to bridge the gap between the specific and the generalized. Do you remember your first exposure to *letters* in math class? Your teacher explained that these letters (variables) were placeholders for numbers, which meant you were now considering the possibility of more than one solution to a problem. That's a big leap and requires a move from concrete thinking to abstract thinking. You may begin to notice this type of generalizing in other areas as well. Instead of thinking only about the here and now, he will begin to reflect on events and make connections that surprise you. For example, when you ask your son how a test went that day, he may respond by saying, "I was really nervous at first, but calmed down when I remembered a relaxation technique my swim coach told me to use just before I stepped onto the starting block." Jean Piaget, the first developmental psychologist to make a systematic study of cognitive development, calls this period from age 11 or 12 into adulthood the Period of Formal Operations.[6] During this stage, a child will demonstrate the ability to critically analyze situations taking into consideration reasoning and argument and demonstrate the ability to think in more abstract terms.

This is one of the reasons why your teacher talks about character traits like maturity,

persistence, and curiosity more than his math skills when you ask about your son's readiness for algebra.

At Haverford, we make this decision very carefully because we know that his success in future math courses hinges on the foundation built in algebra. If a student is not ready to think abstractly, he will resort to memorization, a more comfortable place for a concrete thinker. Later, when his Geometry or Algebra II teacher asks him to use something from Algebra I, he may struggle because he never really learned the material the first time. New learning requires a meaningful connection to previous information in order to become permanent.[7]

If you have a question about your son's readiness for algebra, please ask his teacher, but also keep in mind that our goal is to place every boy in the math course that will appropriately challenge him and ensure long term confidence in his math ability.

Q: Will my son get into college if he does not take Calculus?

A: The answer to this is simply, yes, he will. Although it is understandable why an applicant would want calculus on his transcript when hoping to earn a position at a competitive university, in a College Board post, Bernard Madison of The University of Arkansas argues that a student "should choose calculus only if it is a natural next step in the student's progress. Those who do not intend to pursue science or engineering in college may be better off in AP Statistics or in a course that emphasizes the contextual use of algebra and geometry." [8]

Although more students are completing math courses higher than Algebra II, *U.S. News & World Report* tells us that the most current data on the number of students enrolled in a high school calculus course ranges from 14 to 16 percent.[9]

The college guidance office is your best resource on your son's particular situation, but very few colleges have a definite requirement of calculus or four years of math. A college doesn't want to be in a position where they have to reject an otherwise well-qualified applicant because of a lack of calculus.[10]

The more important question to ask is this: Does my son enjoy math enough to be persistent in this new level, and do his future studies require this high level of math preparation? If algebra represents the first gateway in mathematics, calculus is most certainly the second. Why is calculus so hard? John Buoni, Emeritus Professor of Mathematics & Statistics at Youngstown State University explains:

> "You probably think that calculus is an end of a sequence of courses in mathematics that you arrive at after passing through algebra, geometry, trigonometry, etc. Unfortunately, this is completely incorrect. Calculus is the beginning of a whole new branch of mathematics and the 'right of

> passage' into many areas of the physical, engineering, and social sciences. You must learn a whole new set of tools, and a whole new approach to problem solving involving multiple steps. Calculus is going to teach you a new way of thinking and of looking at mathematics and at nature itself. In calculus, you are going to solve multi-faceted problems unlike previous mathematical studies except geometry." [11]

In summary, calculus requires a mastery of all previous topics in mathematics and if there are gaps, calculus will expose them. If your son has successfully completed a pre-calculus course at The Haverford School with a grade of B or above and has an interest in mathematics, then he is ready to move on to a calculus course. However, this does not mean the transition will be easy. As Buoni asserts, calculus is not simply the next step in mathematics, but a new beginning.

Have a question for the math department?
Please email Math Department Chair Susan Mitchell at *smitchell@haverford.org* or your son's teacher.

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[1] Doyle, Terry and Zakrajsek, Todd. *The New Science of Learning: How to Learn in Harmony with Your Brain*. Sterling Publishing, 2013. Page 11.

[2] *Ibid,* p. 11.

[3] *Ibid,* p. 7.

[4] *Ibid,* p. 81.

[5] Gojak, Linda. *"Algebra: Not if, But When," Summing Up,* National Council of Teachers of Mathematics. 3 Dec. 2013. Web. 1 Sept. 2015.

[6] Piaget, John. "*Stages of Cognitive Development*," *Stages of Intellectual Development In Children and Teenagers*, Child Development Institute. Web. 1 Sept. 2015.

[7] *The New Science of Learning*, p. 7.

[8] Madison, Bernard. *"Mathematics at the School-to-College Transition," AP Central,* College Board. Web. 1 Sept. 2015.

[9] Koebler, Jason. *"American Students Are Taking Harder Math, Science Courses," U.S. News & World Report*. 18 Jan. 2012. Web. 1 Sept. 2015.

[10] *"High School Preparation in Math: Learn How Much and What Level of Math You Need to Get Into College," About.com*. Web. 1 Sept. 2015.

[11] Buoni, John. *"Why is Calculus So Hard?," Youngstown State University*. Web. 1 Sept. 2015.

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Having worked in math education for more than 20 years, Math Department Chair Susan Mitchell is frequently asked her opinion on issues within the field. As a parent of three children, each of whom attended different schools in various states and abroad, she understands the barriers parents face when trying to obtain information about their child's education. Here, she address a few of the most commonly asked questions in math education. We hope it will prompt you to speak with your son or his teachers to learn more about the scope of the mathematics curriculum at The Haverford School.

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